Hydraulic Parameter Estimation

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Getting the most from your geophysical data…

- Three methods to use hydrogeophysical data (Ferre et al., 2009, EOS)
  - Independently
  - Jointly
  - Coupled

- Tomography should be applied cautiously
  - Can resolve subtle subsurface properties
  - But destroys important target geometry

- Proposed analysis only works if you have a notion of the hydrogeological framework
Simple example: surface infiltration

a) surface monitoring

b) borehole monitoring

Resistivity

GPR
Surface monitoring with resistivity

1-D Infiltration

\[ K_{\text{wet}}, \theta_{\text{wet}}, h_{\text{wet}} \]

\[ K_{\text{dry}}, \theta_{\text{dry}}, h_{\text{dry}} \]

2-Layer Resistivity

\[ \rho_{\text{wet}} \]

\[ \rho_{\text{dry}} \]

Green-Ampt Soil Model

For small time, the position of the wetting front is approximated by (Warrick, 2003):

\[
\frac{K_{\text{wet}} t}{\Delta \theta} \approx z_f - \Delta h \left[ \frac{z_f}{\Delta h} - \frac{1}{2} \left( \frac{z_f}{\Delta h} \right)^2 \right]
\]

For large time, the wetting front will approach a constant velocity of:

\[
\frac{dz_f}{dt} = \frac{K_{\text{wet}}}{\Delta \theta}
\]

*note that green = hydraulics equ. yellow = geophysics equ. blue = common variable
Voltages measured over a two-layered earth:

\[ V_{pp}(r) = \frac{I \rho_{\text{wet}}}{2\pi r} G(r, z_f, R) \]

\[ R = \frac{\rho_{\text{dry}} - \rho_{\text{wet}}}{\rho_{\text{dry}} + \rho_{\text{wet}}} \]

\[ G(r, z_f, R) = 1 + 2Rr \int_0^\infty \frac{e^{-2z_f \lambda}}{1 - \text{Re}^{-2z_f \lambda}} J_0(\lambda r) d\lambda \]

\[ G(r, z_f, R) = 1 + 2Rr \sum_{j=0}^\infty \frac{R^j}{\left[ r^2 + \left( (2j + 2) z_f \right)^2 \right]^{1/2}} \]
Coupled Equations

\[ V_{pp}(r, t) = \frac{I \rho_{wet}}{2\pi r} \left[ 1 + 2Rr \sum_{j=0}^{\infty} \frac{R^j}{r^2 + \left(2j + 2\right) \frac{K_{wet}}{\Delta \theta} t} \right]^{\frac{1}{2}} \]
Forward model of voltage potential during infiltration

\[ K_{\text{wet}} = 0.34 \text{ m/hr} \]
\[ K_{\text{dry}} \sim 0 \text{ m/hr} \]
\[ \theta_{\text{wet}} = 0.287 \text{ m}^3/\text{m}^3 \]
\[ \theta_{\text{dry}} = 0.1 \text{ m}^3/\text{m}^3 \]

\[ * \frac{K_{\text{wet}}}{\Delta \theta} = \frac{dz_f}{dt} = 1.8182 \text{ m/hr} \]

Constitutive model for water content to resistivity:

\[ \rho_{\text{soil}} = \rho_{\text{water}} \theta^{-m} \]

\[ m = 2.184 \]
\[ \rho_{\text{water}} = 0.655 \text{ ohm-m} \]
\[ \rho_{\text{wet}} = 10 \text{ ohm-m} \]
\[ \rho_{\text{dry}} = 100 \text{ ohm-m} \]
\[ R = 0.8182 \]

* Remember this value
Time Series Monitoring Results (instrument response)

\[ V_{pp} = \text{voltage for pole-pole array} \]
\[ V_{wen} = \text{voltage for Wenner array} \]
\[ V_{dd} = \text{voltage for dipole-dipole array} \]
**Dimensionless Form**

Vpp = voltage for pole-pole array
Vwen = voltage for Wenner array
Vdd = voltage for dipole-dipole array

*all collapse to an equivalent curve*
Dimensionless Resistivity = \( \frac{\rho_a}{\rho_{wet}} = G(r, z_f, R) \)

Then evaluate kernel at \( z_f = r \) (or \( z_f / r = 1 \)):

\[
G \left( \frac{r}{r}, \frac{z_f}{r}, R \right) = G(1,1, R) = G(R)
\]

where

\[
G(R) = 1 + 2 \sum_{j=0}^{\infty} \frac{R^j}{\left[1 + 4(j+1)^2\right]^{\frac{1}{2}}}
\]
Plot of $G(R)$

- **Wenner**
- **pole-pole**
- **dipole-dipole**

Graph showing $G(R)$ versus $R$ with $G(R)$ on the y-axis and $R$ on the x-axis.
Evaluation of Time Series

- From geometry and voltage measurements, we can get:
  - $r$ and $\rho_{\text{wet}}, \rho_{\text{dry}}, R$

- We want to know:
  - $K_{\text{wet}}, \theta_{\text{wet}}, \theta_{\text{dry}}$

- We can use $G(R)$
G(R) is evaluated when $z_f/r = 1$:

$$\frac{z_f}{r} = \frac{K_{\text{wet}}}{r \Delta \theta}$$

$$K_{\text{wet}} = \frac{r}{\Delta \theta}$$

This is the stupid number we started with!
Finding $K_{\text{wet}}$

- Need to know water content data
  - A priori information about $\rho_{\text{water}}$ and $m$ in
    \[ \rho_{\text{soil}} = \rho_{\text{water}} \theta^{-m} \]

- Cumulative infiltration, $C = \Delta \theta z_f$

- Parameter estimation for unique variables using time series data
  - $K_{\text{wet}}$, $\rho_{\text{water}}$ and $m$
  - Method = nonlinear least-squares inversion
Real Data

Wenner Array

\[ G(R) = 1.22 \]
\[ G(R) = 1.20 \]
Infiltration Monitoring with Borehole Radar

A) Early stage of infiltration

\[ \tau_{direct} = \frac{x}{v_{dry}} \]

Wet
\[ v_l, s_l, \theta_l \]

Dry
\[ v_0, s_0, \theta_0 \]

First arrival travel time

radar antennae

borehole

\[ \tau_{direct} \]
A) Early stage of infiltration

\[ \left\{ \begin{array}{l}
    v_I, s_I, \theta_I \\
    v_0, s_0, \theta_0
\end{array} \right. \]

Wet

Dry

First arrival travel time

\[ \tau_{\text{direct}} = \frac{x}{v_{\text{dry}}} \]

B) Advanced infiltration

Secondary arrivals

\[ \tau_{\text{refr}} = \frac{x}{v_{\text{dry}}} + z_f \left[ \frac{2}{v_{\text{wet}} \cos i_c} - \frac{2 \tan i_c}{v_{\text{dry}}} \right] \]
Borehole Radar

A) Early stage of infiltration

B) Advanced infiltration

C) Late stage of infiltration

\[ \tau_{\text{direct}} = \frac{x}{v_{\text{dry}}} \]

\[ \tau_{\text{refr}} = \frac{x}{v_{\text{dry}}} + z_f \left( \frac{2}{v_{\text{wet}} \cos i_c} - \frac{2 \tan i_c}{v_{\text{dry}}} \right) \]

Constant Flux Model

1-D, water content-based, Richards’ equation

\[ \frac{\partial \theta(z,t)}{\partial t} = \frac{\partial}{\partial z} \left( D(\theta) \frac{\partial \theta}{\partial z} \right) - \frac{dK(\theta)}{d\theta} \frac{\partial \theta}{\partial z} \]

Constant flux top BC, uniform initial water content

\[ t \geq 0, \quad z=0: \quad K(\theta) - D(\theta) \frac{\partial \theta}{\partial z} = R \quad : \text{and} \quad t=0, \quad z>0: \quad \theta = \theta_0 \]

Can be linearized using a Broadbridge & White soil with soil parameters C and S.
**Constant Flux Model**

**Vol. Water Content**

- $w = 0, 0.1, 0.2, 0.3, 0.4, 0.5$
- $f = K_{zw} - W$

**Wetting Front Velocity**

- $t = 4.5 \times 10^4$ s

**Depth of CWR antennae**

- $1.5$

**Equation for Wetting Front Velocity**

$$\frac{dz_f}{dt} = \frac{2(C-1)K_{wet}}{\Delta \theta} \frac{1}{\left[1 + \frac{4K_{wet}C(C-1)}{R} \right]^{1/2} - 1}$$

**But at long times...**

$$\frac{dz_f}{dt} = \frac{K_{wet}}{\Delta \theta}$$
First arrival travel time (FATT):

\[ t_{\text{direct}}(t) = xs_{\text{low}} \quad \text{or} \quad t_{\text{refr}}(t) = xs_{\text{high}} + 2z_f(t)\sqrt{s_{\text{low}}^2 - s_{\text{high}}^2} \]

Taking derivative with time:

\[ \frac{dt_{\text{refr}}}{dt} = 2\sqrt{s_{\text{low}}^2 - s_{\text{high}}^2} \frac{dz_f}{dt} \]

Incorporating wetting front velocity and solve for \( K_{\text{wet}} \):

\[ K_{\text{wet}} = \frac{\frac{dt_{\text{refr}}}{dt}}{\frac{2\sqrt{s_{\text{low}}^2 - s_{\text{high}}^2}}{t}} (\theta_w - \theta_0) \]
Real Data
Real Data

Infiltration time (s) * 10^4

- Pressure (cbars)

TDR Water Content

0.7

0.9

0.11

0.13

0.15

0.17

0.19

0.21

0.0  5  10  15  20  25

0  40  80  120
Real Data

Infiltration time (s) $\times 10^4$

- Pressure (cbars)

- FATT (ns)

- Critical refr direct

TDR Water Content

0.21
0.19
0.17
0.15
0.13
0.11
0.09
0.07
0
0.07
0.09
0.11
0.13
0.15
0.17
0.19
0.21
0
40
80
120
Conclusions

- Simple flow and geophysical models coupled
  - 2-layer resistivity
  - Cross borehole radar
- Theoretical instrument response matched measured data
- Hydraulic properties can be obtained
  - Will provide an average over the measurement volume
  - No assessment of uncertainty if hydro model is different than reality